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WILLIAM W. GUO

ADVANCED MATHEMATICS FOR ENGINEERING AND APPLIED SCIENCES

SECOND EDITION

ALWAYS LEARNING

PEARSON

Advanced Mathematics for Engineering and Applied Sciences

Second Edition

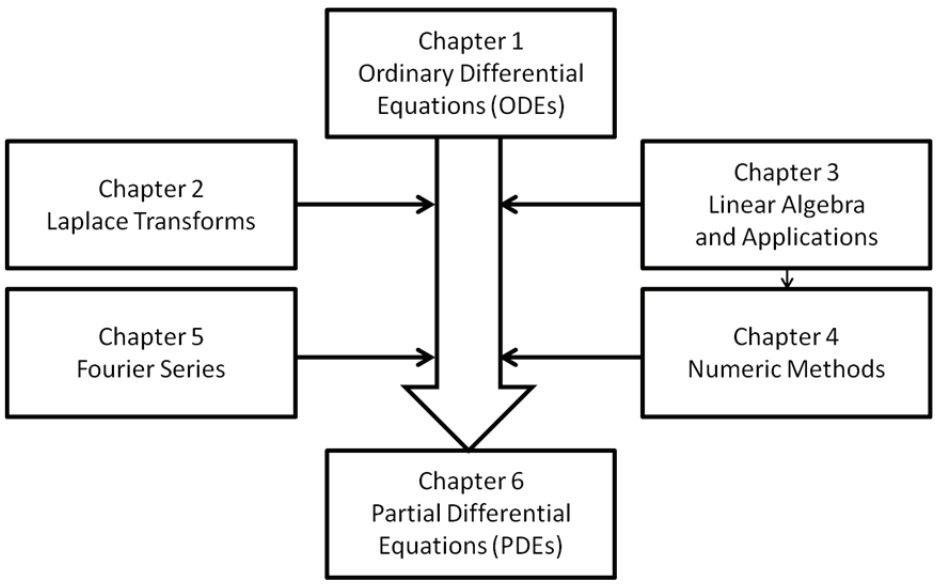
William W. Guo
Central Queensland University Australia

Preface

This book has been written as a designated textbook for university students studying engineering and some areas of applied sciences to continue knowledge building in mathematics after successfully completing a course in elementary calculus. Such an advanced mathematics course needs to cover a wide range of topics of applied mathematics for solving practical problems. For engineering students in US universities, the typical scheme of a comprehensive delivery of such an advanced mathematics course takes four consecutive semesters with at least 4 contact hours per week. Some good textbooks in advanced mathematics used in US universities are also popular in Australian universities for students studying engineering and some areas of applied sciences.

However, the advanced mathematics course in many Australian universities is taught over one semester. Choosing about a quarter of the contents from any US textbook in advanced mathematics for Australian students is always a difficult task in terms of maintaining the continuity and coherence among the chosen topics taught over a single semester. Recently I had direct experience in teaching two large groups of students in engineering mathematics at different levels. The feedback from many students revealed that they were dissatisfied with the discontinuity and incoherence among the chosen topics from the US-oriented textbook. Being a data processing scientist for 30 years of my career whose professional and research activities have been directly or indirectly associated with many areas of advanced mathematics, I took a similar view.

This book is designed to be delivered over one semester of 12–13 teaching weeks with at least 4 contact hours per week. As many scientific and engineering problems are associated with solving ordinary differential equations (ODEs) at an intermediate level and solving partial differential equations (PDEs) at an advanced level, the six chapters in this book together lead students from solving ODEs using various techniques to solving simple PDEs with knowledge and skills gained from the first five chapters. This logic flow is shown in the following diagram.



Based on good practices shared by other authors, my own experiences as an engineering student in the past, and then as a data processing scientist, this book has been written with the following general features:

- A cohesive logical flow that streamlines relevant topics together from solving ODEs to solving PDEs;
- A close connection between mathematical knowledge gained from completing previous math courses to the new topics in this book;
- A gradual increase in the level of difficulty with a smoother transition between two different themes in the same chapter and between two different chapters;
- Loose coupling among the middle chapters (Chapters 2–5) so that the chapters can be delivered almost independently;
- Providing many simple examples to make the book teachable for instructors and understandable for students through self learning;
- Providing real life applications of advanced mathematics in science and engineering.

For a semester-long course, some advanced topics in engineering mathematics, such as 2D/3D Fourier series and Fourier transforms, Bessel functions and Legendre functions, and computer-aided workshops, such as practicing numeric computation using *Matlab* or *Mathematica*, cannot be covered in 12 teaching weeks. Ideally these should be delivered in another subsequent mathematics course.

I am grateful to many engineering students at Central Queensland University who studied engineering mathematics courses in the 2013 academic year. It is the students' strong desire to learn mathematics, their active engagement with the teaching and learning processes, their earnest effort on both individual and group assignments, and their trust in my ability to assist them in achieving the best possible learning outcomes that inspired me to write this designated textbook in advanced mathematics for Australian engineering and science students. Their every endeavour deserves our special attention and full support as educators. Feedback, comments and suggestions on this first edition from students and other readers are most welcome and much appreciated. Special thanks go to the Customs Team at Pearson Australia for their great assistance to make the book published and available in a very short period of time.

Proof reading was done during the Christmas and New Year period by Harry, my son who has recently graduated with a Bachelor of Laws and a Bachelor of Commence from The University of Western Australia. His knowledge in laws offered little help in a mathematical context, but his advice brought changes to my writing style from 'scientific writing' for scientists towards 'plain writing' for junior undergraduate students. His criticism of my frequent use of long and complex sentences led to the significant reduction of such sentences in the final version. This book was written mostly on weekends and holidays during the past six months. I am deeply grateful to my wife Anna for her whole-hearted support throughout the entire journey.

William Guo
January 2014

A Note for Second Edition

It was a great success in improving student learning outcomes and experience by adopting the first edition of this textbook for the second-year engineering students in Semester 1 of 2014 at Central Queensland University Australia. The overwhelming positive feedback from so many students made this course "2014 CQUniversity Student Voice Commendation". This honour is also a reflection of the tremendous effort on achieving quality teaching made by the teaching team during the semester.

Many students and colleagues have helped find out numerous typos in mathematical formulas, texts, tables, and diagrams since the book was released in February 2014. These found typos have been corrected in this new edition. According to the feedback from students through various communications, students like the structure, coverage of contents, and extensive use of worked examples throughout the text. As a result, this new edition keeps these aspects unchanged.

A number of students suggested leaving more spare spaces in the book so that students can take important notes nearby the key concepts or examples during a lecture, a tutorial, or even watching the recorded videos. Other students suggested using a colour scheme to better differentiate different (sub)sections and themes that were presented in the first edition using a simple black-white scheme. These two suggestions have been partly adopted in this new edition by both using dividing lines before or after a (sub)section/theme or an example and providing a spare space immediately after a (sub)section/theme or an example wherever possible. To keep the price of the book low, a grey scheme for (sub)headlines and dividing lines is used, instead of a colour scheme.

Two Appendixes are also included in this new edition to provide students with references to differentiations and integrations respectively.

Many students also expressed an interest in studying another mathematical course/unit after completing this one. It would be much better to have another elective mathematical course/unit for the students to keep advancing mathematical knowledge and problem solving skills in various engineering applications. This can only happen by the time when electives are made available in a new engineering curriculum. A few students suggested me writing a new mathematical textbook tailored to our first-year engineering students, just like this tailored book for our second-year students. This is certainly a great suggestion but also means a huge commitment. We might be able to make this happen in the future driven by our "can-do" approach.

My sincere appreciation goes to my students and colleagues at Central Queensland University for their encouragement and support all the time. This new version was done mostly on weekends in the past ten months. Once again I am very grateful to my wife Anna for her whole-hearted support throughout seemingly an endless journey.

Professor William Guo
School of Engineering & Technology
Central Queensland University Australia
December 2014

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